

A HYBRID FULL-WAVE ANALYSIS OF VIA HOLE GROUNDS USING FINITE DIFFERENCE AND FINITE ELEMENT TIME DOMAIN METHODS

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ABSTRACT

A hybrid full-wave analysis using FDTD(finite difference time domain) and FETD(finite element time domain) methods has been developed to analyze microwave devices with locally arbitrarily shaped three-dimensional structures. This method is applied to calculate the scattering parameters of cylindrical via hole grounds. The comparison of the results with the mode matching data and FDTD staircasing data verifies the accuracy of this analysis.

I. Introduction

The FDTD(finite difference time domain) method has a number of advantages in analyzing three-dimensional microwave structures due to its simplicity in formulation and implementation. Making use of the uniform mesh in the conventional FDTD algorithm does not require any special mesh generation scheme and storage for the mesh. However, the FDTD analysis has difficulty in dealing with curved structures and needs very fine mesh in the entire computational domain for locally detailed structures. Several methods have been developed to overcome these difficulties, including the graded mesh FDTD algorithm [1], the FDTD algorithm in curvilinear coordinates [2], the nonorthogonal FDTD algorithm [3], the locally conformed FDTD algorithm [4], and so on.

Recently, a hybrid method was developed to model the locally curved structure by incorporating the FEM(finite element method) into the FDTD [5]. This method utilizes the advantage of the more flexible FEM while retaining

all the advantages of the FDTD. However, the FDTD and FEM mesh matching method in the interface region introduces difficulty in the mesh generation of the FEM region.

This paper proposes a new FDTD and FETD hybrid method by introducing an interpolation scheme for communicating between the FDTD field and the FETD field. In this method, we avoid the effort of fitting the FETD mesh to the FDTD cells in the interface. This hybrid method has been applied to analyze a via hole grounded microstrip, which is a three-dimensional problem having both cylindrical and rectangular boundaries. Applying the FETD to the via hole grounds and the FDTD elsewhere preserves the advantages of both FDTD and FETD. The comparison of the scattering parameter results with mode matching data [6] and FDTD staircasing data verifies the accuracy of this analysis.

II. Hybrid analysis technique using FDTD and FETD

This hybrid analysis employs the standard FDTD method with Super absorbing Mur's 1st order ABC (absorbing boundary condition) and the FETD method based on the vector finite element method using the prism element.

FETD formulation

Starting from the source-free Maxwell's two curl equations in a linear isotropic region, the vector wave equation can be obtained as

$$\nabla \times \nabla \times \vec{E} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0. \quad (1)$$

Applying the weak form formulation, or the Galerkin's procedure to (1) gives

$$\begin{aligned} \int (\nabla \times \vec{E}) \cdot (\nabla \times \vec{W}) dv + \int \mu\epsilon \vec{W} \cdot \frac{\partial^2 \vec{E}}{\partial t^2} dv \\ = \int \vec{W} \cdot (\nabla \times \vec{E} \times \hat{n}) ds, \quad (2) \end{aligned}$$

where \vec{W} is the testing function defined using the 2nd order prism element [7].

The prism element is composed of 30 edge elements as shown in Fig. 1. Since the trial function is chosen to be the same as the testing function, the electric field inside the prism element can be interpolated as

$$\begin{aligned} \vec{E}(r, t) = & \\ & \sum_{l=1}^3 \sum_{i=1}^3 L_i \frac{l_i}{\Delta} (b_i \hat{z} + c_i \hat{x}) f_{yl} \mathcal{E}_{te}(t)_{i+6(l-1)} \\ & - \sum_{i=1}^3 L_j \frac{l_i}{\Delta} (b_i \hat{z} + c_i \hat{x}) f_{yl} \mathcal{E}_{te}(t)_{i+3+6(l-1)} \\ & + \sum_{l=4}^5 \sum_{i=1}^3 L_i (2L_i - 1) f_{yl} \mathcal{E}_{ye}(t)_{i+6(l-1)} \\ & + \sum_{i=1}^3 4L_i L_j f_{yl} \mathcal{E}_{ye}(t)_{i+3+6(l-1)} \hat{y}, \quad (3) \end{aligned}$$

where

$$\begin{aligned} f_{y1} &= L_{y1}(2L_{y1} - 1) = f_{y4} \\ f_{y2} &= 4L_{y1}L_{y2} \\ f_{y3} &= L_{y2}(2L_{y2} - 1) = f_{y5} \quad (4) \end{aligned}$$

and L_i 's are the barycentric coordinates of a triangle, and L_{yi} 's are the first order Lagrange interpolation polynomials between upper and lower triangular faces.

In the finite differencing of (2) in time, the unconditionally stable backward difference has been used, which gives:

$$\begin{aligned} & \left[\int (\nabla \times \vec{W}) \cdot (\nabla \times \vec{W}) dv + \frac{\mu\epsilon}{\delta t^2} \int \vec{W} \cdot \vec{W} dv \right] \mathcal{E}^n \\ &= \mathcal{E}^n \int \vec{W} \cdot (\nabla \times \vec{W} \times \hat{n}) ds \\ &+ \frac{\mu\epsilon}{\delta t^2} \left[2\mathcal{E}^{n-1} \int \vec{W} \cdot \vec{W} dv - \mathcal{E}^{n-2} \int \vec{W} \cdot \vec{W} dv \right], \quad (5) \end{aligned}$$

where \mathcal{E}^n represents the electric field at the time step n.

Hybridizing the FDTD analysis and the FETD analysis

The r.h.s. of (5), the load vector of the FETD shows the necessity of knowing the electric field values for one and two time steps ahead, as well as the boundary values at the present time step. The FETD region in the FDTD region is chosen to be a brick replacing the part of the FDTD region and includes the interesting structures of arbitrary shape. The FDTD region is valid up to one layer inside of the FETD region.

Fig. 2 shows the FDTD and FETD interface region. In the FDTD time-marching procedure, $Ey^n(i, j, k - 1)$, $Ez^n(i, j, k)$, and $Ez^n(i, j - 1, k)$ are updated, but the FDTD boundary value, $Ey^n(i, j, k)$ can not be updated because $Hx^{n-\frac{1}{2}}(i, j, k + 1)$ does not exist. The updated interface values are employed in the FETD through the boundary integral of (5) to calculate the inner field in the FETD domain. The calculated inner field is used to update the FDTD boundary value, $Ey^n(i, j, k)$. Once $Ey^n(i, j, k)$ is updated, the $Hx^{n+\frac{1}{2}}(i, j, k)$ can be calculated from H field updating procedure of FDTD. This completes the one time-marching procedure of the hybrid method.

This approach makes use of an interpolation scheme in communicating between the FETD field and the FDTD field in the interface. Fig. 3 explains this communication method. From FETD to FDTD, the FETD field is interpolated using the FETD prism element shape function (3). On the other hand, the FETD interface field can be interpolated using the brick element shape function with the FDTD interface field. This scheme can be more general than the FDTD and FETD mesh matching method.

III. Numerical Results

This hybrid method has been employed to characterize the cylindrical via hole grounds in microstrip. The via hole region is replaced by the

FETD region as shown in Fig. 4. Fig. 5 shows the cross-sectional view of the FETD mesh. In simulating the via hole with 0.6 mm diameter, $10 \times 8 \times 10$ cells are replaced by FETD region among the total $120 \times 40 \times 140$ FDTD cells. Since the microstrip and the ground plane coincide with the top and the bottom boundaries of the FETD region, the Dirichlet boundary conditions are applied to the top and the bottom as well as the via hole cylinder region, which reduces the matrix size in the FETD analysis and increases the computational efficiency.

Fig. 6 compares the calculated scattering parameters of a via hole grounded microstrip using this method and two different stircasings in the FDTD analysis (Fig. 7). The via hole is approximated using the inner bounded stircasing in model 1, and the outer bounded stircasing in model 2. As expected, the hybrid method predicts the middle value between those of the two stircasing models. Furthermore, the calculated results are compared with the published mode matching data [6] in Fig. 8. The results show good agreement with the mode matching data in both 0.6 mm diameter and 0.8 mm diameter via hole cases.

IV. Conclusion

A hybrid full-wave analysis using FDTD and FETD methods has been developed and applied to successfully characterize via hole grounded microstrips. The interpolation scheme employed in this analysis alleviates the effort of fitting the FETD mesh to the FDTD cells in the interface region. With this hybrid method, three-dimensional locally curved and detailed structure can be analyzed accurately and efficiently.

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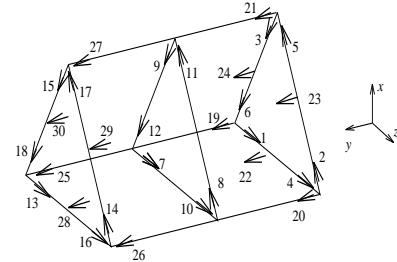


Figure 1: The prism element.

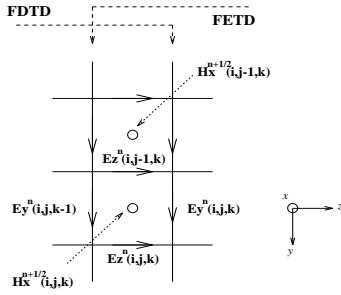
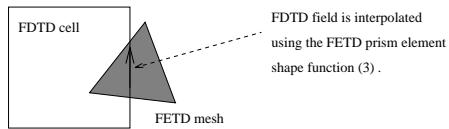


Figure 2: The FDTD and FETD interface region.

A. FETD \Rightarrow FDTD



B. FDTD \Rightarrow FETD

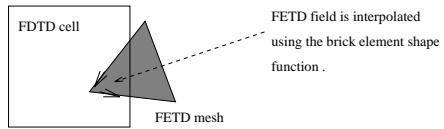


Figure 3: Communication between FDTD and FETD field.

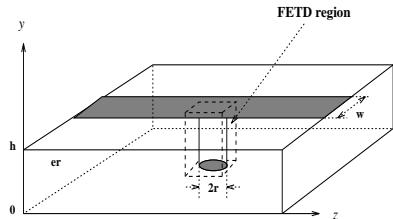


Figure 4: The via hole grounded microstrip.

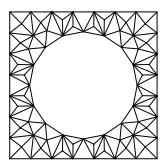


Figure 5: The cross-sectional view of FETD mesh around the via hole.

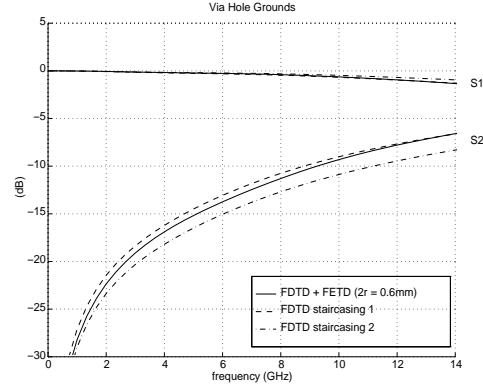


Figure 6: $|S_{11}|$ and $|S_{21}|$ of the via hole grounded microstrip

$(\epsilon_r = 2.32, w = 2.3\text{mm}, h = 0.794\text{mm})$

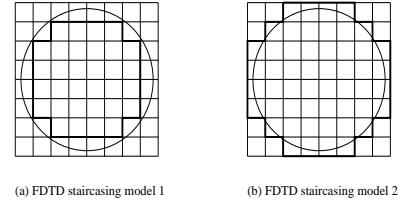


Figure 7: Two staircasing models of the FDTD analysis.

$(dx = dz = 0.082143\text{mm})$

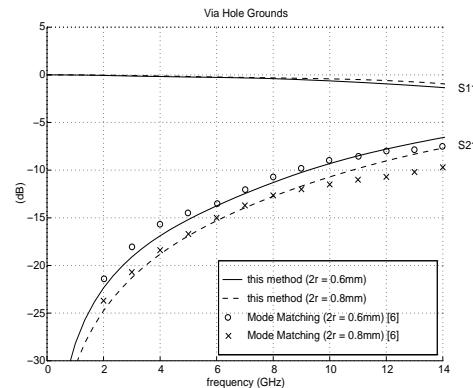


Figure 8: Comparison of the calculated $|S_{21}|$ with the mode matching data.

$(\epsilon_r = 2.32, w = 2.3\text{mm}, h = 0.794\text{mm})$